DEFINITION OF THE VERTICAL FROM LAGEOS LASER RANGING

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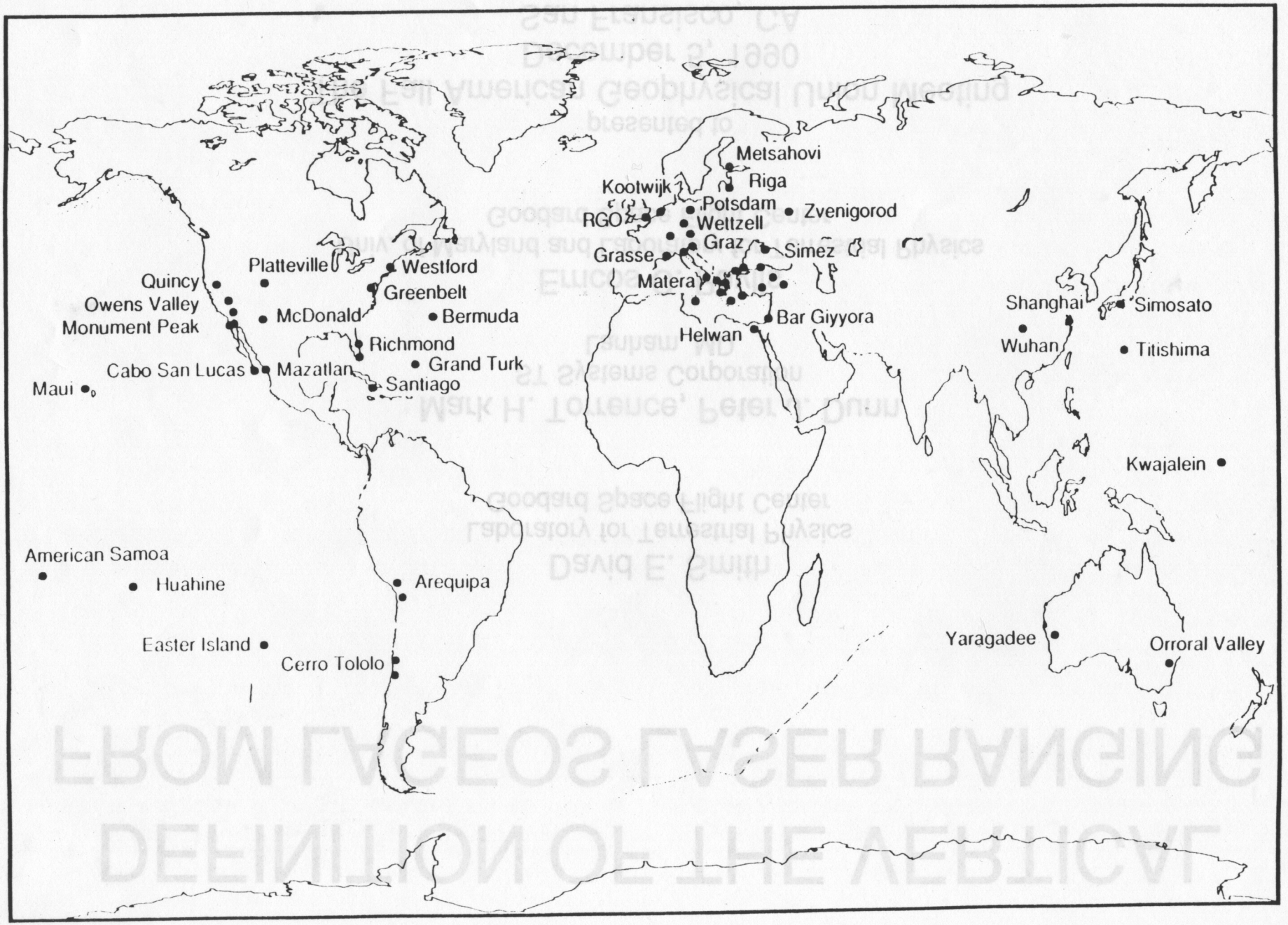
LOCATIONS OF SLR SITES



SL7.1 SOLUTION CONSTANTS AND PARAMETERS

Data extends from May 1976 to June 1990

533,144 two minute normal points

GEM-T1 Geopotential model

Earth and ocean tides from GEM-T1

Third body perturbations from Sun, Moon, and Mercury through Neptune

Along track acceleration and solar radiation coefficients estimated every 15 days

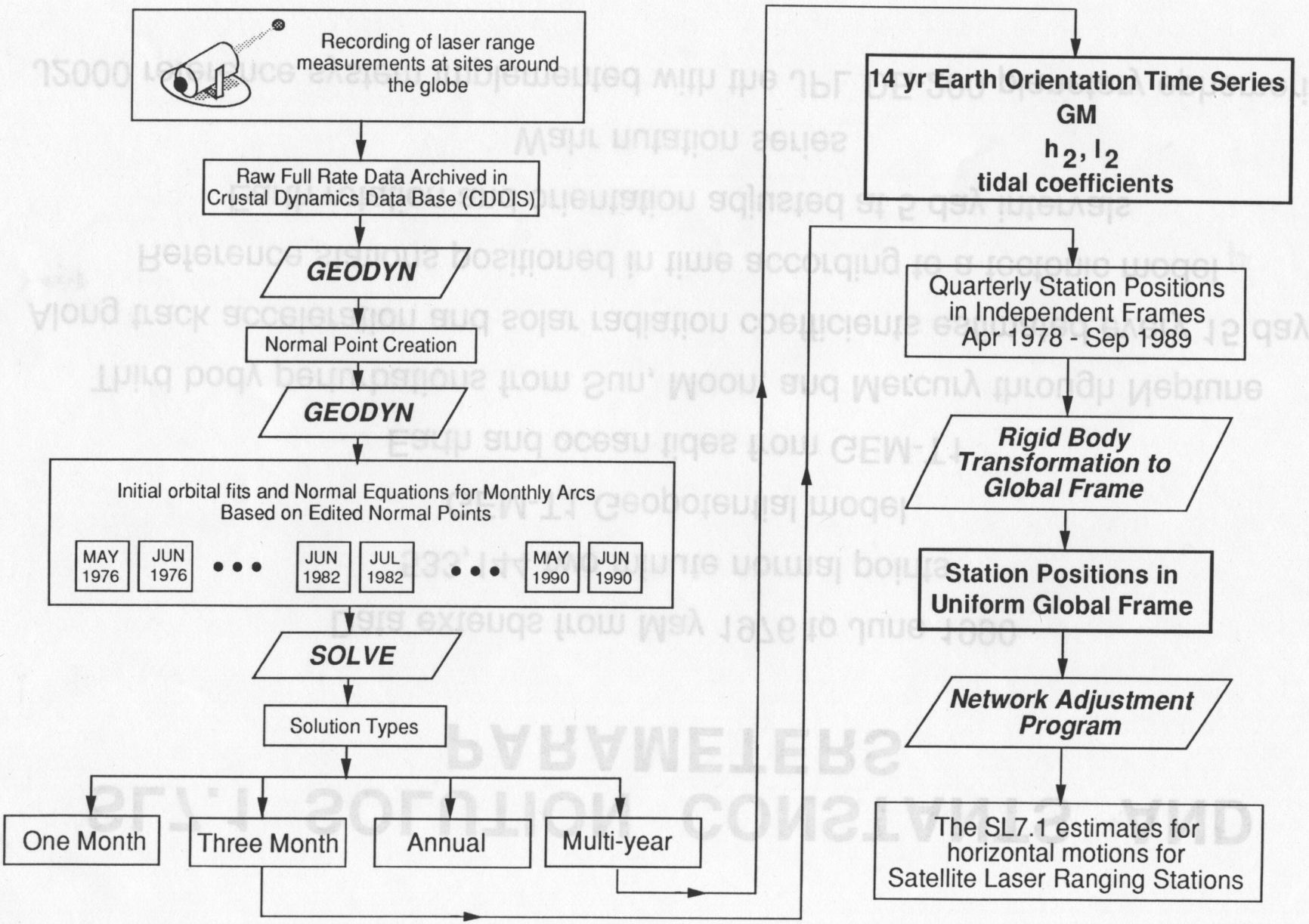
Reference stations positioned in time according to a tectonic model

Earth rotation and orientation adjusted at 5 day intervals

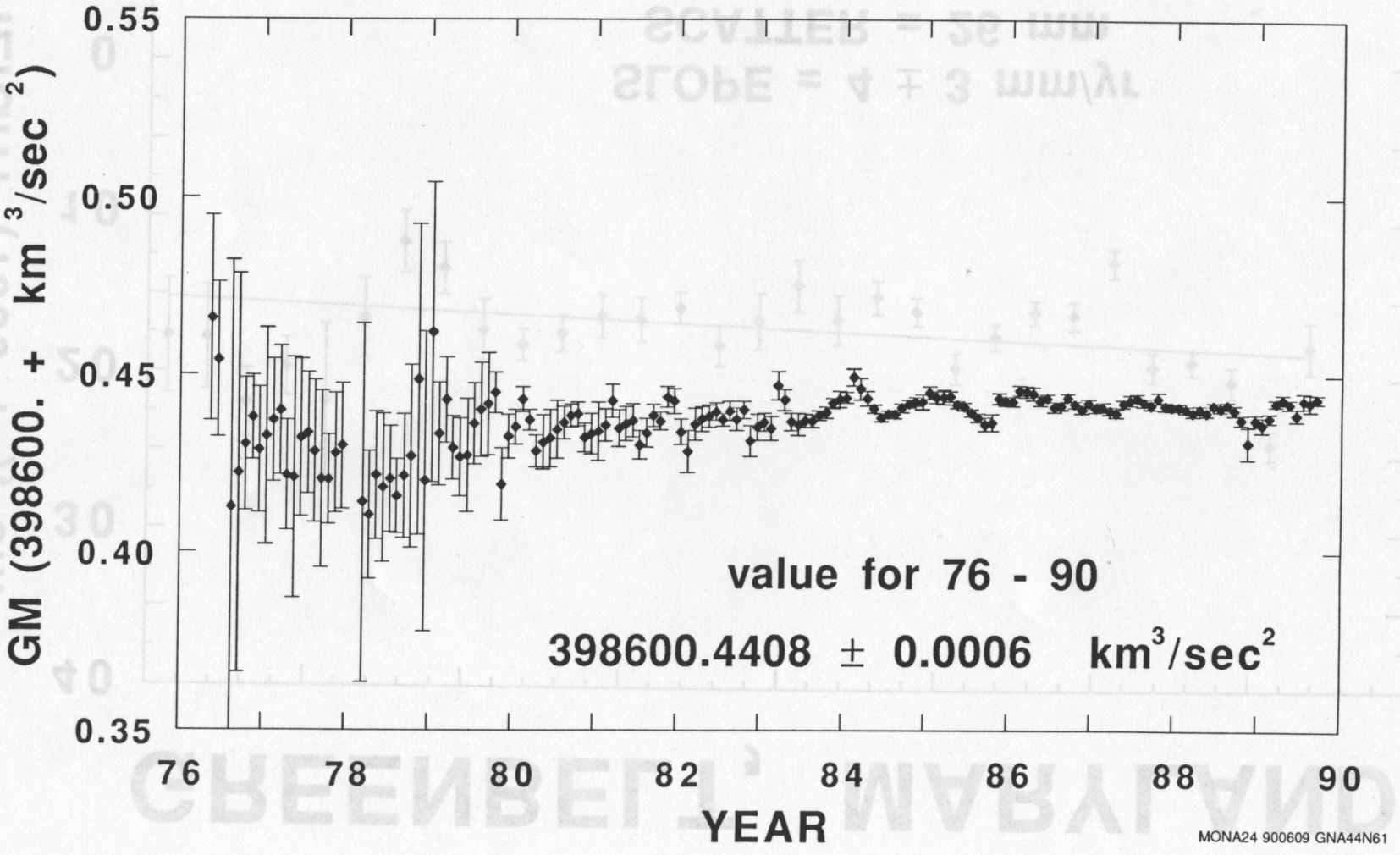
Wahr nutation series

J2000 reference system implemented with the JPL DE-200 planetary ephemeris

SL7.1 SOLUTION CONCEPTS

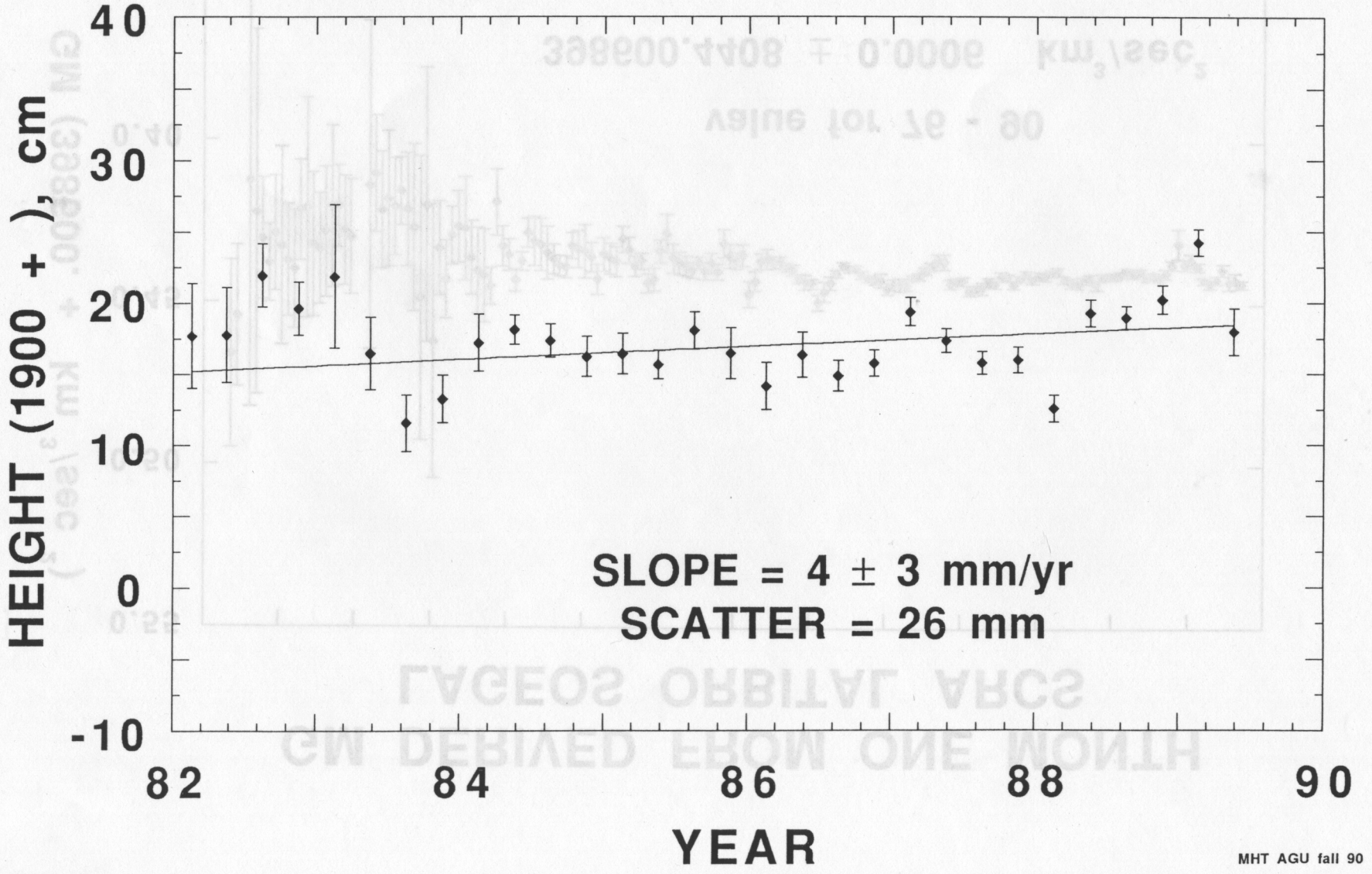


GM DERIVED FROM ONE MONTH LAGEOS ORBITAL ARCS



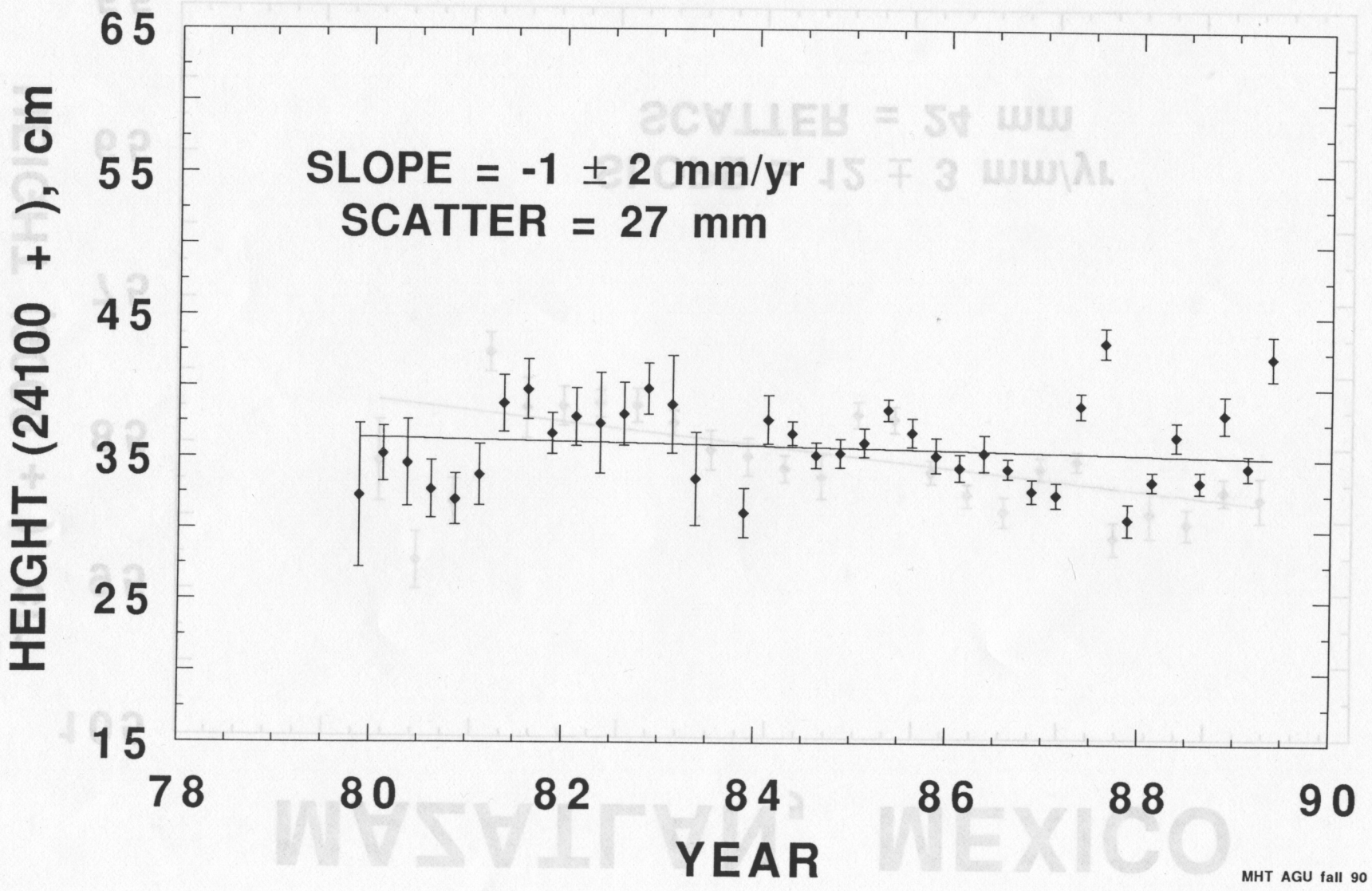
MONA24 900609 GNA44N61

GREENBELT, MARYLAND



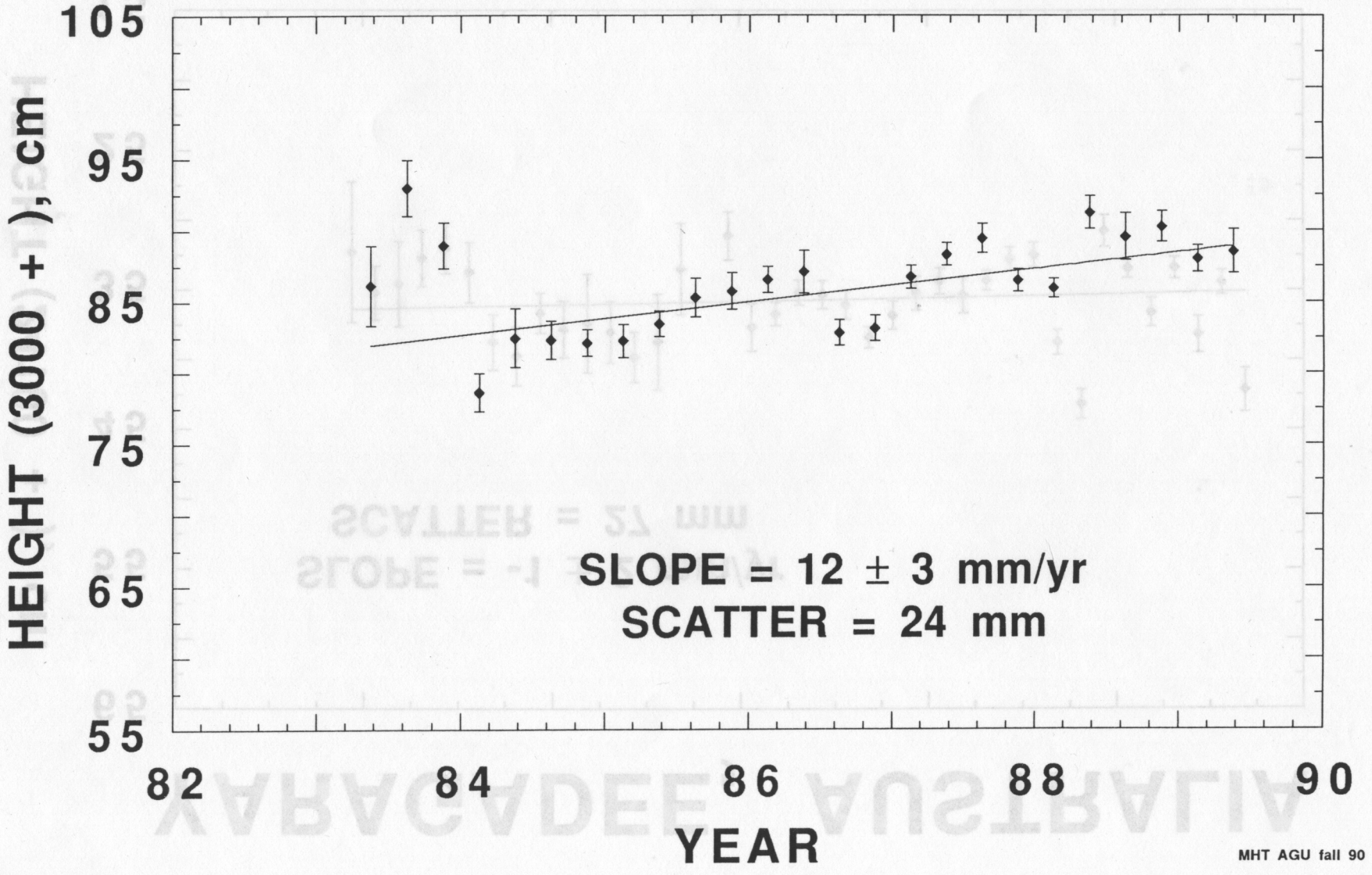
MHT AGU fall 90

YARAGADEE, AUSTRALIA



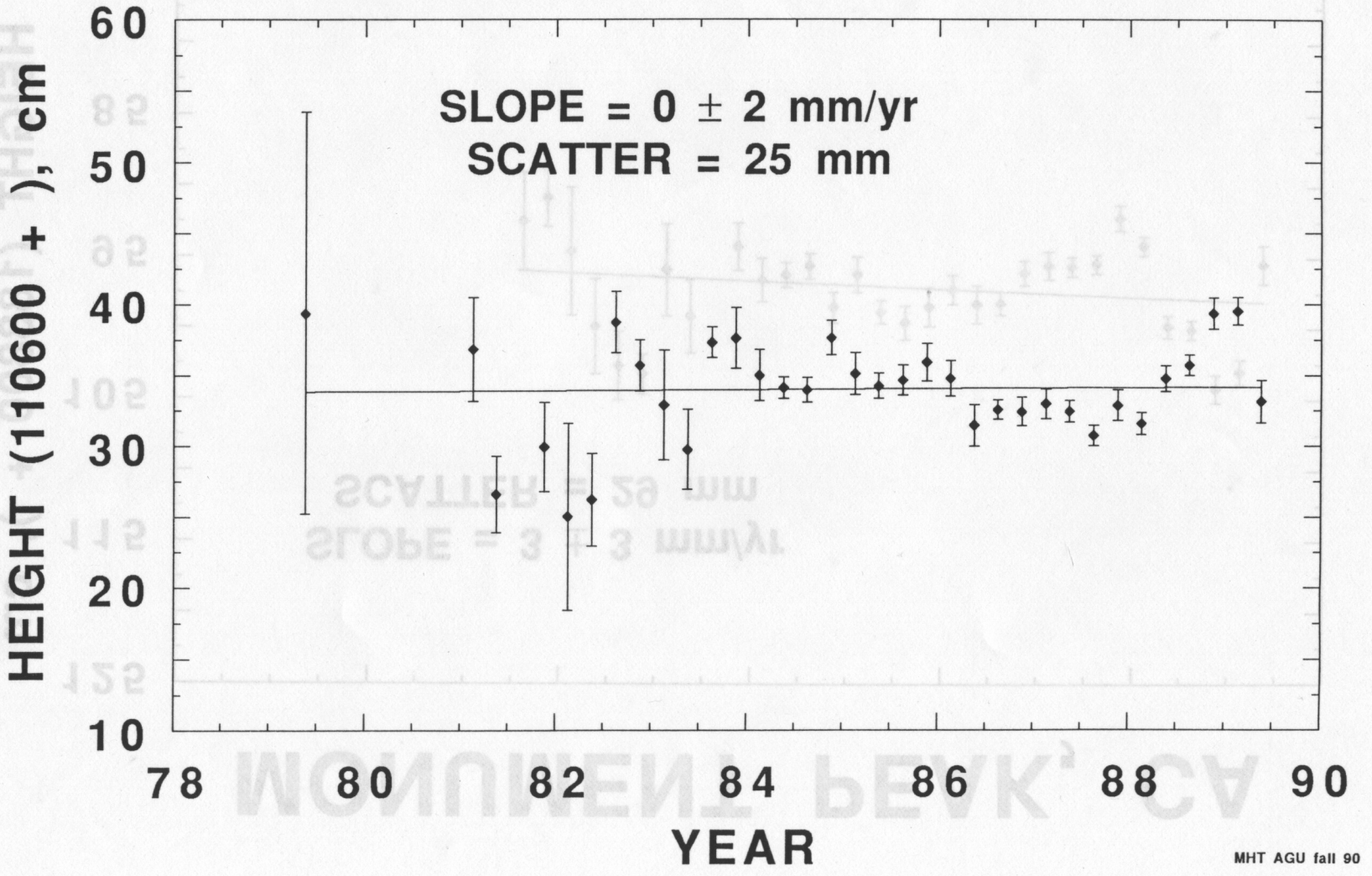
MHT AGU fall 90

MAZATLAN, MEXICO



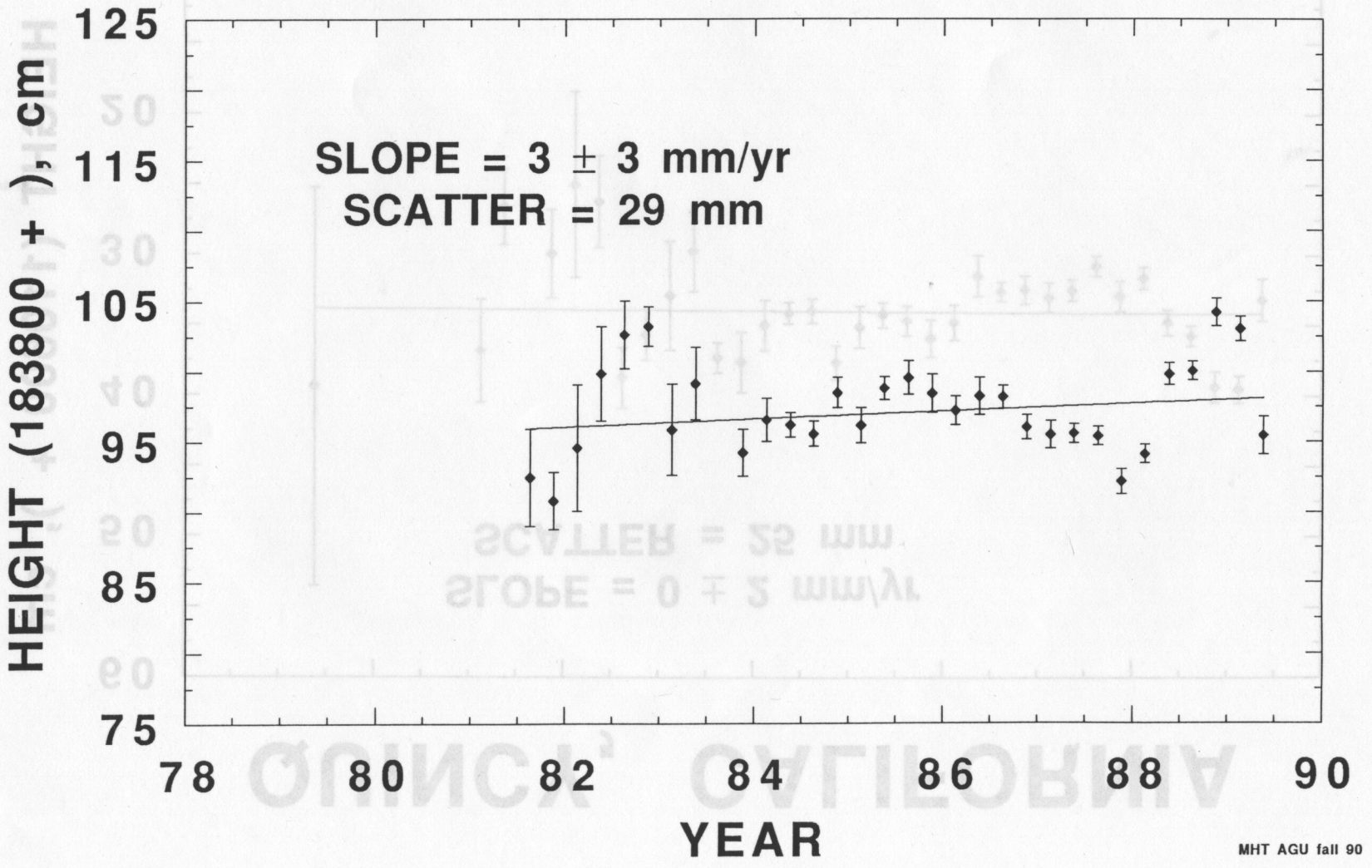
MHT AGU fall 90

QUINCY, CALIFORNIA



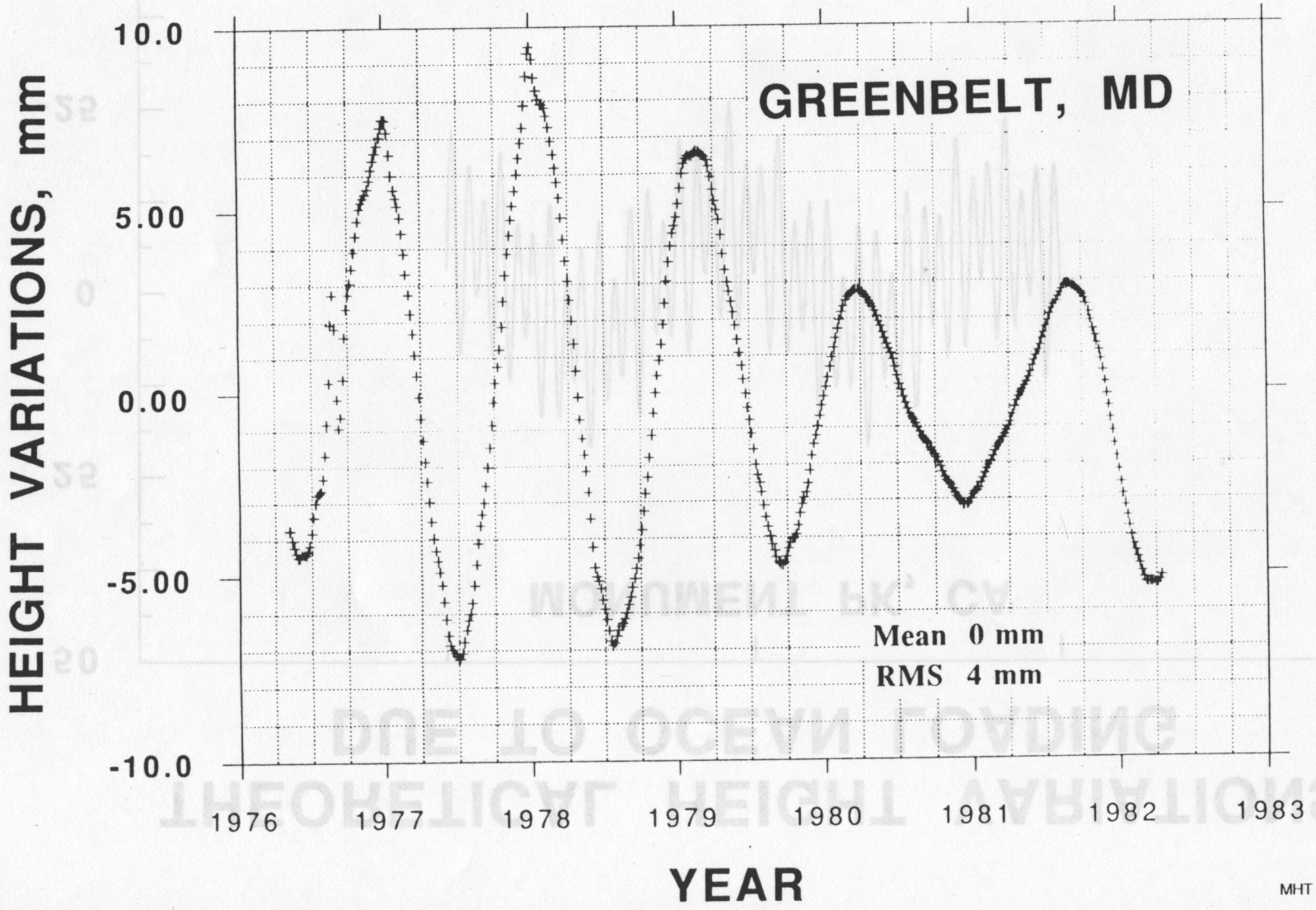
MHT AGU fall 90

MONUMENT PEAK, CA

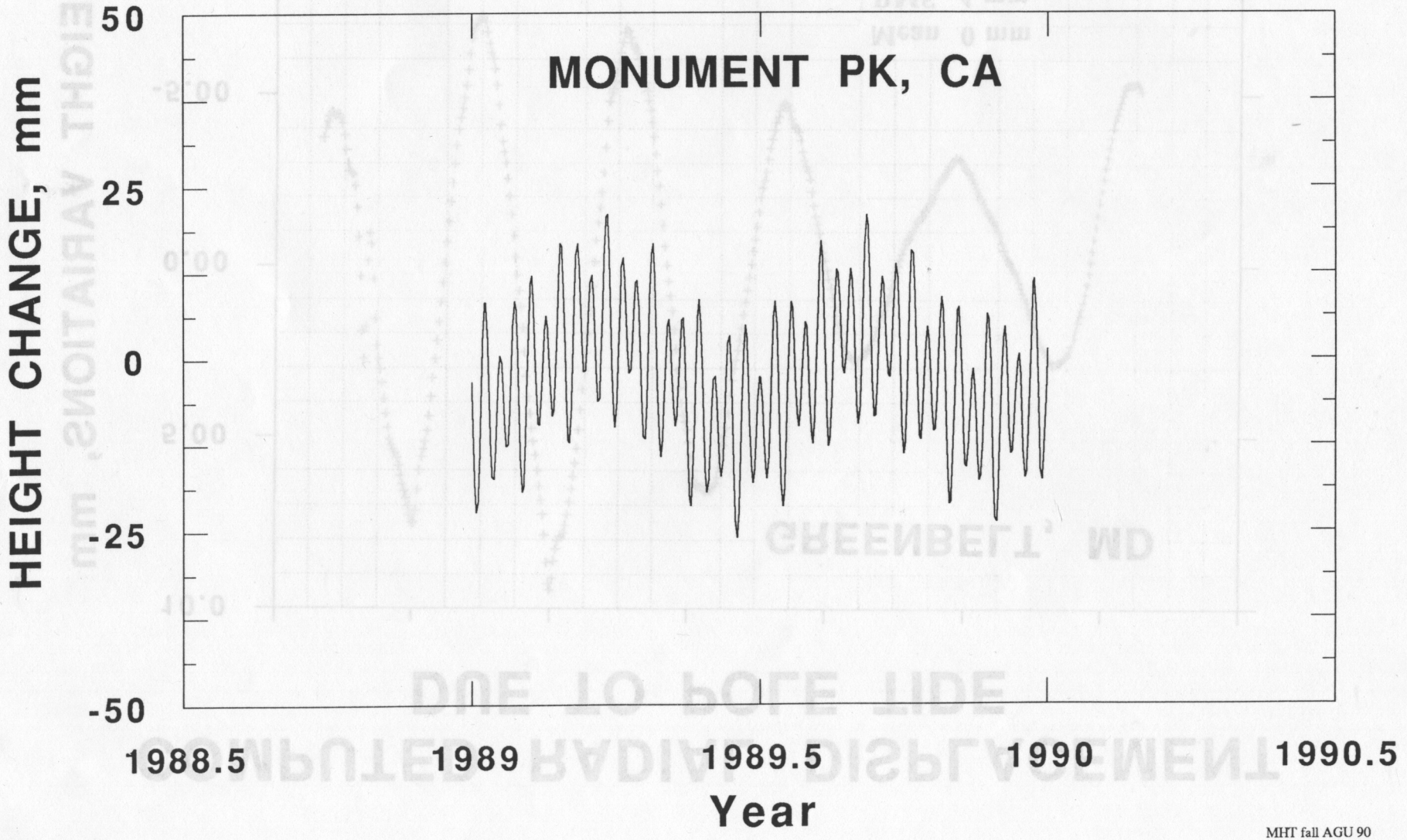


MHT AGU fall 90

COMPUTED RADIAL DISPLACEMENT DUE TO POLE TIDE

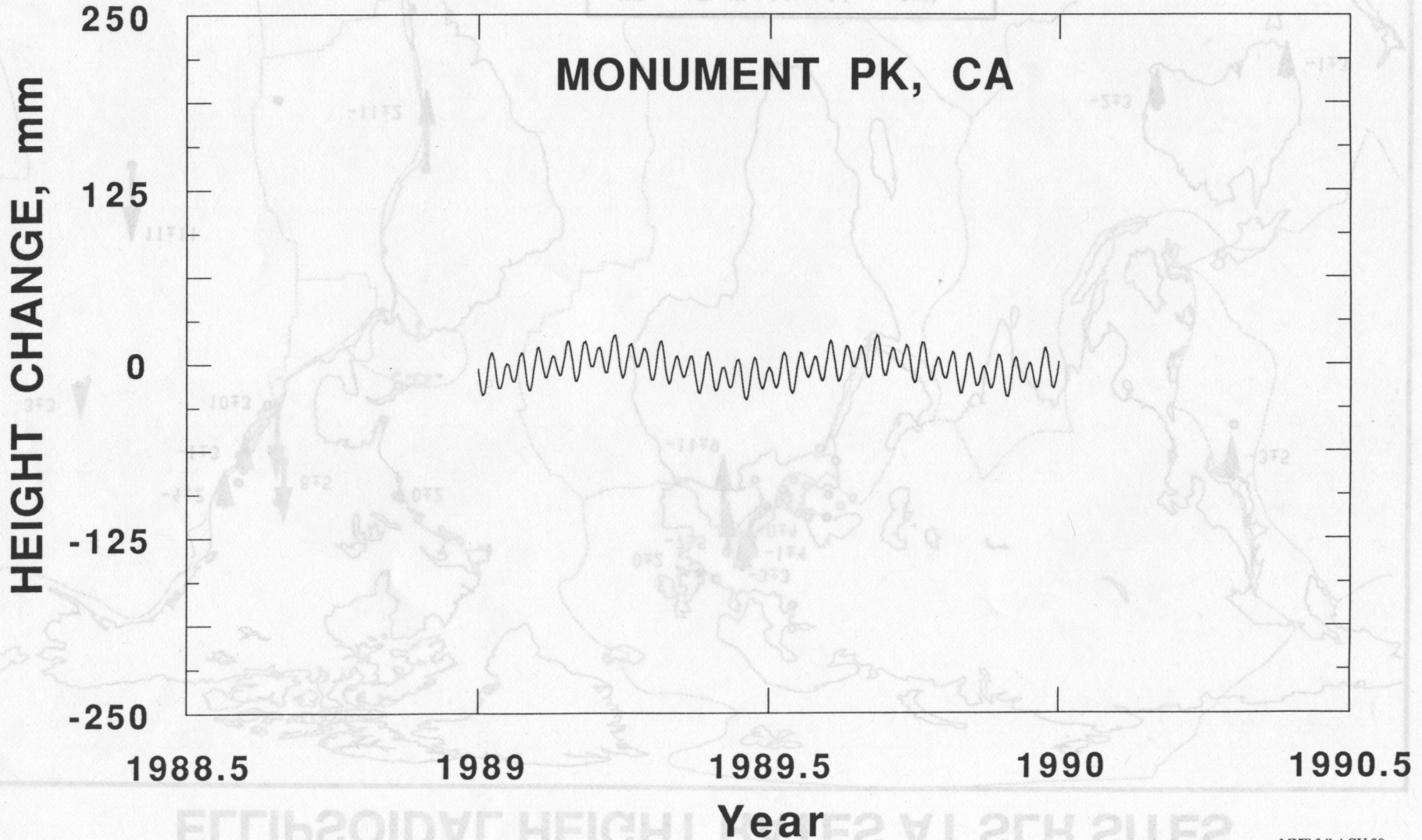


THEORETICAL HEIGHT VARIATIONS DUE TO OCEAN LOADING



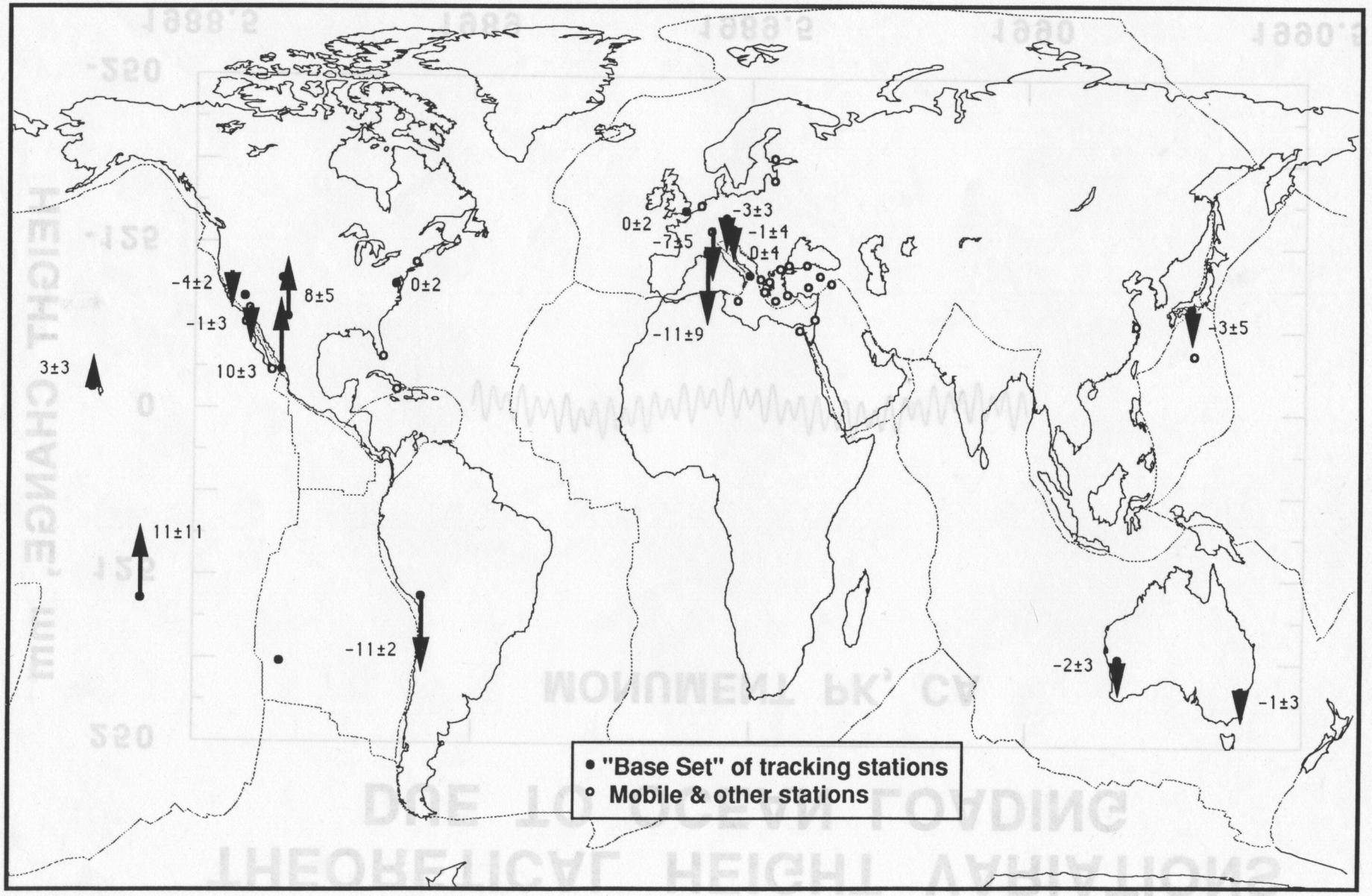
MHT fall AGU 90

THEORETICAL HEIGHT VARIATIONS DUE TO OCEAN LOADING



MHT fall AGU 90

ELLIPSOIDAL HEIGHT RATES AT SLR SITES



NASA/GSFC

MHT AGU Fall 90 QRA8FAN1

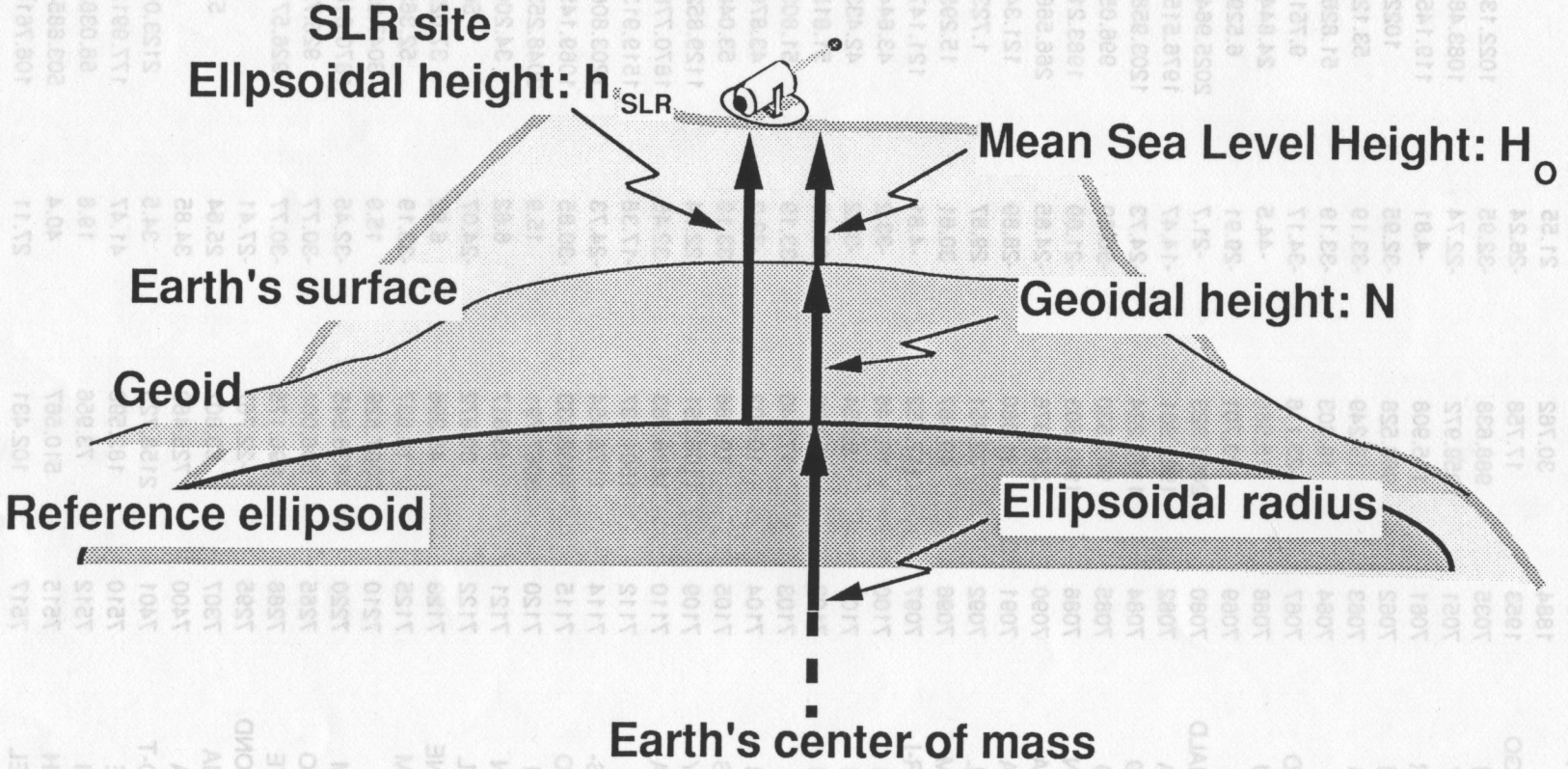
ELLIPSOIDAL HEIGHTS FROM SL7.1

Site Location	number		Epoch = 1984 Height (m)	Height rate (mm/yr)	rms scater
POTSDAM	1181	23	146.818 ± 0.020	-30.3 ± 8.6	72.8
RIGA	1884	2	53.090 ± 0.476		
OTAY MOU	7035	7	987.938 ± 0.023	-10.4 ± 13.1	53.2
QUINCY *	7051	34	1059.002 ± 0.004	0.4 ± 2.4	24.7
EASTER I	7061	5	114.432 ± 0.034	-94.6 ± 23.9	27.1
OTAY MOU	7062	6	987.756 ± 0.026	-40.7 ± 13.8	32.1
GREENBEL	7063	11	18.774 ± 0.013	-37.6 ± 18.6	34.5
PATRICK	7069	2	-23.671 ± 0.027		
MCDONALD	7080	31	2004.091 ± 0.005	15.2 ± 10.0	26.0
BEAR LAK	7082	5	1962.281 ± 0.072		
MCDONALD	7086	26	1961.049 ± 0.007	8.2 ± 3.7	30.4
YARAGADE	**7090**	**38**	**240.992 ± 0.004**	**-1.3 ± 2.0**	**26.6**
WESTFORD	7091	7	91.056 ± 0.015	9.4 ± 10.3	34.2
KWAJALEI	7092	4	30.823 ± 0.027		
AMERICAN	7096	5	47.012 ± 0.035		
EASTER I	7097	8	117.259 ± 0.040	43.1 ± 13.1	63.9
GREENBEL	7102	12	17.181 ± 0.021	30.2 ± 15.8	58.9
GREENBEL	7103	3	17.116 ± 0.007	19.3 ± 5.9	2.9
GREENBEL	7105	42	19.019 ± 0.004	3.2 ± 2.1	26.7
QUINCY	**7109**	**34**	**1106.002 ± 0.004**	**0.4 ± 2.4**	**24.7**
MONUMENT	**7110**	**32**	**1838.016 ± 0.005**	**2.7 ± 3.1**	**29.5**
PLATTEVI	7112	17	1500.742 ± 0.022	-43.0 ± 13.9	69.7
OWENS VA	7114	9	1176.806 ± 0.015	-32.3 ± 6.3	19.6
GOLDSTON	7115	7	1037.704 ± 0.015		
HALEAKAL	7120	7	3067.401 ± 0.014	66.8 ± 16.4	16.2
HUAHINE	7121	12	43.163 ± 0.012	27.1 ± 14.5	34.3
MAZATLAN	**7122**	**25**	**30.070 ± 0.006**	**11.6 ± 3.3**	**23.5**
HUAHINE	7123	17	45.107 ± 0.020	17.9 ± 16.2	76.0
GREENBEL	7125	2	18.391 ± 0.006		
HALEAKAL	7210	36	3067.041 ± 0.006	6.9 ± 2.6	30.7
MONUMENT	7220	2	1838.744 ± 0.015		
MOJAVE	7288	3	896.036 ± 0.010	6.0 ± 6.4	10.4
RICHMOND	7295	2	-22.862 ± 0.016		
SANTIAGO	7400	2	725.117 ± 0.002		
ASKITES	7510	3	181.939 ± 0.012		
KATAVIA	7512	4	73.043 ± 0.008		

ELLIPSOIDAL HEIGHTS FROM SL7.1

Site Location	number		Epoch = 1984 Height (m)	Height rate (mm/yr)	rms scater
DIONYSOS	7515	3	509.926 ± 0.006	-12.4 ± 14.0	6.6
ROUMELLI	7517	5	102.025 ± 0.010	4.2 ± 11.7	18.5
XRISOKAL	7525	4	475.597 ± 0.022		
BAR GIYY	7530	6	773.977 ± 0.012		
LAMPEDUS	7544	3	116.151 ± 0.448		
PUNTA SA	7545	2	229.001 ± 0.000		
YOZGAT	7585	2	1681.406 ± 0.043		
YIGILCA	7587	2	821.608 ± 0.007		
MONTE GE	7590	2	1650.267 ± 0.047		
METSAHOV	7805	16	74.771 ± 0.315		
ZIMMERWA	7810	17	949.956 ± 0.008	-7.3 ± 5.3	29.9
HELWAN	7831	4	131.280 ± 0.052		
KOOTWIJK	7833	17	93.179 ± 0.025	29.9 ± 14.6	89.0
WETTZELL	7834	34	659.998 ± 0.006	-0.3 ± 3.0	31.6
GRASSE	7835	25	1321.623 ± 0.008	-62.9 ± 24.2	33.2
SHANGHAI	7837	8	26.950 ± 0.021	-8.3 ± 12.8	53.1
SIMOSATO	7838	28	98.989 ± 0.007	-1.9 ± 4.5	34.8
GRAZ	7839	23	539.029 ± 0.007	4.9 ± 4.5	31.9
HERSTMON	7840	24	75.019 ± 0.003	3.1 ± 2.2	15.4
ORRORAL	7843	22	1348.495 ± 0.047		
OWENS VA	7853	10	1177.919 ± 0.014	-13.5 ± 6.1	33.2
CABO SAN	7882	3	111.365 ± 0.080		
QUINCY	7886	2	1109.127 ± 0.013		
MOUNT HO	7888	2	2331.949 ± 0.019		
FLAGSTAF	7891	2	2143.965 ± 0.001		
VERNAL	7892	3	1589.576 ± 0.030		
YUMA	7894	2	241.809 ± 0.009		
GREENBEL	7899	2	5.518 ± 0.051		
AREQUIPA	7907	44	2491.947 ± 0.005	-8.9 ± 1.7	23.9
MOUNT HO	7921	3	2352.399 ± 0.065		
NATAL	7929	7	39.223 ± 0.015		
MATERA	7939	24	535.016 ± 0.007	2.7 ± 4.2	30.7
ORRORAL	7943	13	948.022 ± 0.008	3.7 ± 13.0	28.1
KOOTWIJK	8833	4	88.077 ± 0.027		
	number		70	38	
			average	9.2	34.4

DETERMINATION OF EARTH'S SEMIMAJOR AXIS



$$\Delta A_e = h_{SLR} - H_o - N$$

EARTH'S SEMIMAJOR AXIS

name	num	Ellipsoidal Height	Geoid Height	Mean Sea Level	Delta Ae	
POTSDA	1181	147.847	40.66			
RIGA	1884	30.762	21.55			
SANTIAGO	1953	17.758	-26.24			
OTAY-M	7035	988.638	-32.95	1022.13	-0.54	
QUINCY	7051	1059.972	-22.74	1083.48	-0.77	
EASTER	7061	115.908	-4.81	119.145	1.57	
OTAY-M	7062	988.528	-32.95	1022	-0.52	
STALAS	7063	19.249	-33.19	53.12	-0.68	
GORF	7064	18.603	-33.19	51.826	-0.03	
BERMUD	7067	-23.116	-34.17	9.751	1.30	
GND-TU	7068	-18.566	-44.5	24.844	1.09	
PTRK-A	7069	-23.701	-29.91	6.529	-0.32	
MCDONALD	7080	2004.323	-21.7	2025.964	0.06	
BEARLA	7082	1962.901	-14.47	1976.515	0.86	
OVRO19	7084	1178.224	-24.73	1203.958	-1.00	
GLDS19	7085	965.530	-30.69	996.05	0.17	
MC-DON	7086	1961.503	-21.69	1983.21	-0.02	
YARAGA	7090	241.376	-24.85	266.568	-0.34	
HAYSTA	7091	91.980	-28.89	121.34	-0.47	
KWAJEL	7092	32.951	29.87	1.723	1.36	
AM-SOM	7096	49.067	30.61	15.295	3.16	E
EASTER-I	7097	117.552	-4.81	121.147	1.22	
GORF-1	7100	10.346	-33.2	43.644	-0.10	
GORF-1	7101	8.654	-33.2	42.433	-0.58	
GORF-1	7102	17.993	-33.19	51.815	-0.63	
ML0601	7103	17.940	-33.19	51.803	-0.67	
GORF-1	7104	10.002	-33.2	43.876	-0.67	
GORF-S	7105	19.186	-33.19	53.048	-0.67	
QUINCY	7109	1106.351	-22.74	1129.852	-0.76	
MN-PEA	7110	1838.983	-32.45	1870.778	0.65	
PLATVI	7112	1501.537	-17.38	1519.913	-1.00	
OWENS-	7114	1178.054	-24.73	1203.806	-1.02	
GLDSTO	7115	1038.633	-30.85	1069.145	0.34	
HAWAII	7120	3067.791	15.9	3048.253	3.64	E
HUAHIN	7121	43.617	6.62	34.206	2.79	E
MAZATL	7122	30.872	-24.07	56	-1.06	
HUAHINE	7123	45.396	6.62	37.023	1.75	
GORF-M	7125	18.537	-33.19	52.363	-0.64	
LURE	7210	3067.526	15.9	3047.95	3.68	E
TLRS-M	7220	1838.845	-32.45	1870.618	0.68	
BARSTO	7265	896.004	-30.77	926.72	0.05	
MOJAVE	7288	896.175	-30.77	926.577	0.37	
RICHMOND	7295	-22.767	-27.41			
TITISIMA	7307	83.807	25.64	57	1.17	
SANTIA	7400	725.460	34.85			
CERRO-T	7401	2158.725	34.5	2123.09	1.14	
ASKITE	7510	182.598	41.47	177.9913	-36.86	E
KATAVI	7512	73.956	19.8	68.0383	-13.88	E
MT1ATH	7515	510.567	40.4	503.8854	-33.72	E
ROUMEL	7517	102.431	27.11	106.7618	-31.44	E

EARTH'S SEMIMAJOR AXIS

	name	num	Ellipsoidal Height	Geoid Height	Mean Sea Level	Delta Ae	
	KARITS	7520	598.625	33.41	601.77	-36.55	E
	XRISOK	7525	476.199	23.3	476.9253	-24.03	E
	BARGIO	7530	774.435	20.65	751.72	2.07	E
	MT2MAT	7541	528.378	45.71	528.7508	-46.08	E
	LMPADU	7544	111.918	36.84			
	CAGLIA	7545	229.983	46.93	232.1879	-49.13	E
	MEDICINA	7546	49.477	40.72			
	BASOVI	7550	447.103	44.63	439.0041	-36.53	E
	DIYARB	7575	724.958	24.37			
	MLNGIC	7580	1357.782	34.06			
	YOZGAT	7585	1677.134	34.61	1684.1417	-41.62	E
	YIGILC	7587	822.700	35.33			
	MTGNRO	7590	1648.440	48.41	1663.4743	-63.44	E
	MT-WTZ	7596	657.330	46.86	650.746	-40.28	E
	FINLAN	7805	77.837	19.38	59.23	-0.77	
	ZIMRWA	7810	951.032	49.97			
	UNKNOW	7811	122.482	35.82			
	HELWAN	7831	131.879	15.51			
	KOOTWI	7833	93.443	44.13	49.8	-0.49	
–	WETTZE	7834	661.134	46.86	653.2	-38.93	E
–	GRASSE	7835	1322.878	51.88	1281.15	-10.15	E
	SHANGH	7837	27.848	10.95	-29.023	45.92	E
–	SIMOSA	7838	99.453	38.18	60.34	0.93	
–	GRAZ	7839	539.419	47.95	495.427	-3.96	E
–	GRENWI	7840	75.390	45.95	30.68	-1.24	
–	ORRORAL	7843	1350.006	18.58	1345.934	-14.51	E
	LAMPADUS	7844	262.835	47.23	213	2.60	E
	OWENS-V	7853	1178.129	-24.73			
	CABO-S	7882	111.316	-32.61			
	ENSENADA	7883	165.010	-32.94	200	-2.05	E
	FT-DAVIS	7885	1961.359	-21.69	1983.161	-0.11	
	TLRS-Q	7886	1109.620	-22.74			
	VANDBU	7887	601.367	-35.77	636.45	0.69	
	TLRS-H	7888	2331.329	-29.36	2334.45	26.24	E
	AUSTIN	7890	257.330	-25.99	244.6	38.72	E
	FLAGST	7891	2144.320	-22.47	2167.1	-0.31	
	VERNAL	7892	1590.296	-16.38	1593.26	13.42	E
	YUMA	7894	241.806	-32.9			
	PASADE	7896	441.661	-33.5	474.886	0.28	
	GORF78	7899	10.053	-33.2	43.885	-0.63	
	AREQUI	7907	2492.285	47.01	2450.963	-5.69	E
	MT-HOP	7921	2352.774	-29.36	2383.387	-1.25	
	NATAL	7929	39.698	-4.83	45.6	-1.07	
	MATERA	7939	535.855	45.71	490.52	-0.38	
	ORRORA	7943	949.053	18.61	929.53	0.91	
	MTLRKO	8833	88.637	44.14	88.5977	-44.10	E

Average Ae (from 79 values) **6378131.52 ± 17.85 m**

The average Ae (edited to 50 values) **6378136.96 ± 0.83 m**

SUMMARY

- GM = 398600.4408 ± 0.0006 km^3/sec^2
- Linear rates for ellipsoidal heights determined at the 3-5 mm/yr level
- The Earth's semimajor axis A_e 6378137.0 ± 0.8 m